# Minnesota State High School Mathematics League

## Individual Event

## 2006-07 Event 5A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have **20** minutes for this event.

1. 1. Two line segments 2 inches long and three line segments 1 inch long are arranged in Figure 1A to form two squares of side length 1. Show in the space provided for Figure 1B how to arrange four line segments 2 inches in length and four line segments 1 inch long to form three squares of side length 1.

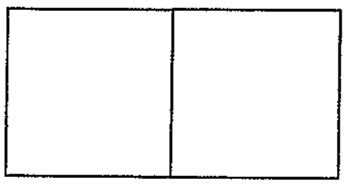


Figure 1A

Figure 1B

2. Figure 2 shows the integers 1, 2, 3, 4, 5, 6 placed at points along a triangle so that the sum of the integers on each side is 12. On the remaining triangles, place the same six integers so that the sums along each side of a triangle are equal, but equal to a different sum for each triangle.

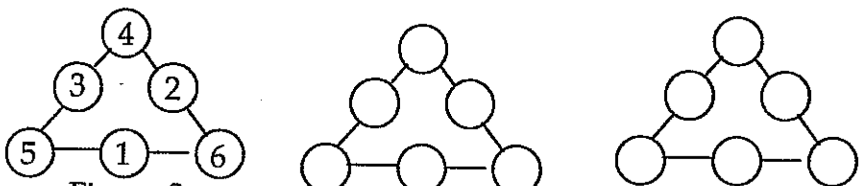
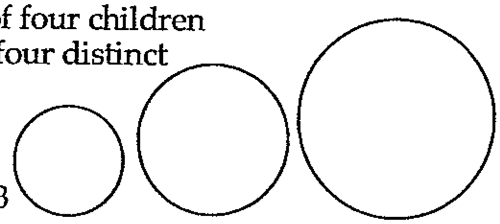



Figure 2

3. The ice cream pies shown in Figure 3 are uniformly thick and have radii of 3, 4, and 5. The pies are to be divided so that each of four children get the same amount. Show on the drawings how to make four distinct cuts, one a circle, and the other three straight lines, that accomplish the goal. Label with A, B, C, and D the parts to be given to each child.

Figure 3



________ 4. Ashley, Betty, Carlos, Dick, and Elgin went shopping. Each had a whole number of dollars, and together they had \$56. The absolute difference between the amounts Ashley and Betty had was \$19. The absolute difference between the amounts Betty and Carlos had was \$7; between Carlos and Dick was \$5; between Dick and Elgin was \$4; between Elgin and Ashley was \$11. How much did Elgin have?

Name____________________________Team____________________________

# Minnesota State High School Mathematics League

## Individual Event

## 2006-07 Event 5B

**The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.**

________ 1. The rhombus *ABCD* pictured in Figure 1 has sides of length 4, and the altitude dropped from *D* bisects *AB*. What is the area of *ABCD*?

________ 2. The mid-points of the rhombus described in Problem 1 are joined to form an inscribed quadrilateral. What is its area ?

3. In the square *ABCD* with sides of length 1 shown in Figure 3, let $x = AE = BF = CG = DH$. Find the area of *RSTU* in terms of $x$.

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4. Using the square and the notation introduced in Problem 3, find the area of $\Delta BCS$ in terms of $x$.

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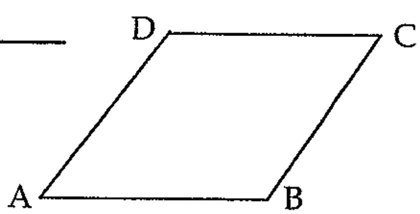


Figure 1

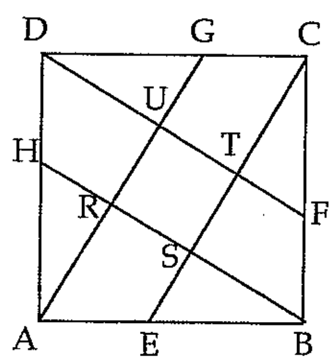


Figure 3

Name______________________________Team____________________________

# Minnesota State High School Mathematics League

## Individual Event

## 2006-07 Event 5C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_______ 1. In the expansion of $(x+y)^{15}$, what is the coefficient of $x^9y^6$?

_______ 2. The Screening Committee is to present to the President a list, in alphabetical order, of the names of three acceptable candidates. How many different lists are possible, given that the names of the candidates are: Abdullah, Bjorn, Costillo, Diem, Ellsworth, Fiorello, Gin, Hu, Ione, Juarz, Klein, Longley.

_______ 3. A bag contains 4 red and 8 white marbles, well mixed. One marble is removed and replaced by two marbles of the other color. Again after thorough mixing, a marble is drawn. What is the probability that this last marble to be removed is red?

_______ 4. If $a$, $b$, and $c$ are three (not necessarily different) integers chosen randomly and with replacement from the set {1. 2. 3. 4. 5}, what is the probability that $ab+c$ is even?

Name_______________________________ Team_______________________

# Minnesota State High School Mathematics League

## Individual Event

## 2006-07 Event 5D

Questions in this event are written, with permission, as variations of problems from the 2006 AMC-12 Exam. Review of this exam is excellent preparation, not only for this event, but for the 2007 AMC-12, which we strongly encourage you to take.

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

__________ 1. For any two real numbers $x$ and $y$, define $x \oplus y = \frac{x+y}{2}$. Find the numeric value of $[(3 \oplus 5) \oplus 7] - [3 \oplus (5 \oplus 7)]$.

__________ 2. The vertices of a 5-12-13 right triangle are centers of three mutually externally tangent circles as shown in Figure 2. What is the sum of the areas of these circles?

__________ 3. The function $f$ has the property that for each real number x in its domain, $\frac{1}{x}$ is also in its domain, and $f(x) + f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$. What is the largest set of numbers that can be in the domain of $f$?

__________ 4. The expression $(x+y+z)^{2007} + (x-y-z)^{2007}$ is expanded and simplified by combining like terms. How many terms are in this simplified expression?

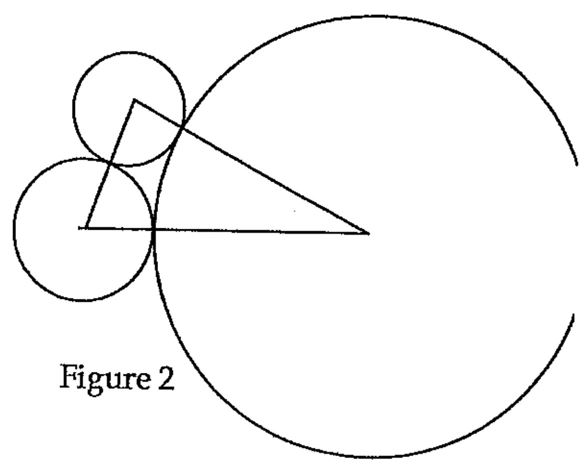

Figure 2

Name__________________________ Team__________________________

# Minnesota State High School Mathematics League

## Team Event

## 2006-07 Meet 5

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. In Figure 1, place the integers 1, 2, 3, 4, 5, 6, 7, 8 in the circles along the edges of the square so that the sum of the integers on each side is the same. Do the same in each of the remaining squares, taking care to get sums that are the same on each side of a square, but different for each square. You get 1 point for each square completed.

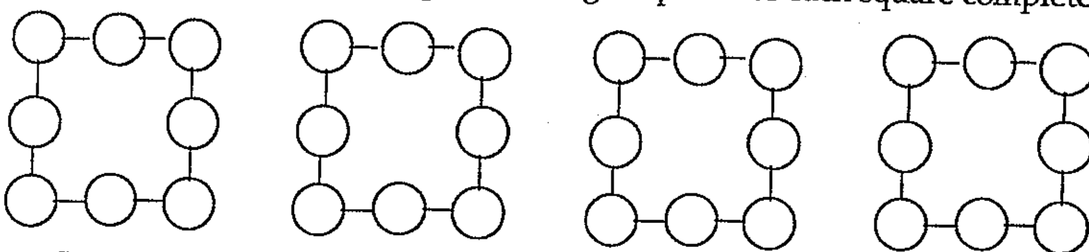

Figure 1

2. Four points on a plane determine six line segments. Figure 2 shows one way to place the four points so that the six line segments have just two different lengths. You get one point, up to a maximum of four, for each essentially different way (simply rotating or reflecting a figure doesn't count) you can place four points so that the six connecting line segments have just two different lengths. Clearly indicate the given four points, since the lines may cross in places not included among the given points, as in Figure 2.

3. To begin with, a student walks down a row of lockers, numbered from 1 to 500, making sure that they are all closed but not locked. Student 2 walks along the lockers and changes the position (in this case opens) all the doors with numbers divisible by 2. Student 3 then changes the position of the doors of lockers with numbers divisible by 3 (opening number 3, closing number 6, etc.). This process continues, with student $n$ changing the position (opening closed doors, closing open doors) of doors on lockers with numbers divisible by $n$.

________ (a) What is the final position of door 360 (open or closed)?

________ (b) How many times will door 360 have been opened when the process is complete?

________ 4. A quadrilateral is formed by joining in order $O(0,0)$ to $A(16,0)$ to $B(8,14)$ to $C(2,10)$, and back to $O$. The midpoints of the sides of OABC are then joined in order to form a second quadrilateral. What is its area?

5. In the square $ABCD$ with sides of length 1 shown in Figure 5, let $x = AE = BF = CG = DH$. Find the area of $\Delta BSE$ in terms of $x$.

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6. Let $x$ be the length of the base of an isosceles triangle having legs of length 10. We wish to study the function $f$ that expresses the area $A$ of the triangle as a function of $x$; i.e. $A = f(x)$.

________ (a) (2 points) Write a formula for $f(x)$

________ (b) (1 point) What is the domain of this function?

________ (c) (1 point) What is the value of $x$ that maximizes $A$?

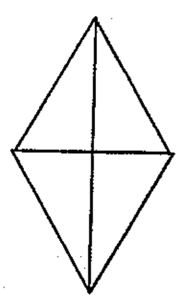

Figure 2

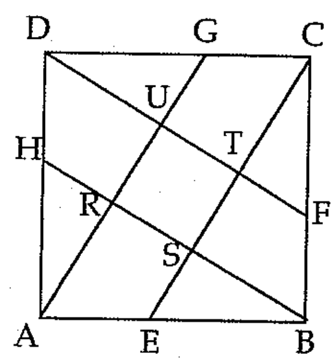


Figure 5

Put your drawings for problem 2 here.

Team________